Let $x = r\cos\theta$, $\dfrac{r^2(a^2-b^2)}{b^2(a^2-r^2)} = e^2$, $\dfrac{r^2}{a^2-r^2} = c$.

$$V = \frac{4br^2\sqrt{(a^2-r^2)}}{a}\int_0^{\frac{1}{2}\pi}\sqrt{(1-e^2\sin^2\theta)}\sin^2\theta d\theta$$

$$+\frac{4br^2\sqrt{(a^2-r^2)}}{3a}\int_0^{\frac{1}{2}\pi}\frac{\cos^2\theta d\theta}{\sqrt{(1-e^2\sin^2\theta)}}$$

$$+\frac{8abr^2}{3\sqrt{(a^2-r^2)}}\int_0^{\frac{1}{2}\pi}\frac{\cos^2\theta d\theta}{(1+c\sin^2\theta)\sqrt{(1-e^2\sin^2\theta)}}.$$

$$\therefore\ V = \frac{8abr^2(c+1)}{3c\sqrt{(a^2-r^2)}}\Pi(e, c, \tfrac{1}{2}\pi) + \frac{8br^2(a^2-r^2)}{3a}E(e, \tfrac{1}{2}\pi)$$

$$+\frac{4br^2}{3ce^2\sqrt{(a^2-r^2)}}[c(1+e+e^2)(a^2-r^2)-2a^2e^2]\,F(e, \tfrac{1}{2}\pi)$$

$$= \frac{8a^3b}{3\sqrt{(a^2-r^2)}}\Pi(e, c, \tfrac{1}{2}\pi) + \frac{8br^2(a^2-r^2)}{3a}E(e, \tfrac{1}{2}\pi)$$

$$+\frac{4b\sqrt{(a-r^2)}}{3ae^2}[r^2(1+c+e^2)-2a^2e^2]\,F(e, \tfrac{1}{2}\pi).$$

Also solved by *J. SCHEFFER*.

MECHANICS.

87. Proposed by H. C. WHITAKER. M.E., Ph.D., Professor of Mathematics, Manual Training School, Philadelphia. Pa.

"He on his impious foes onward drove,
Drove them before him to the bounds
And crystal walls of Heaven; which opening wide
Rolled inward and a spacious gap disclosed
Into the wasteful deep; headlong themselves they threw
Down from the verge of Heaven.
Nine days they fell; Hell at last
Yawning received them whole and on them closed."

Paradise Lost, Book VI.

Assuming Hell to be the center of the earth, and the only force acting on the lost spirits to be that of gravity due to the earth's attraction,—How far is Heaven ?

II. Solution by the PROPOSER.

The published solution of No. 87, Mechanics, assumes (page 82, line 4) that $s = \frac{1}{2}gt^2$ is the formula for falling bodies for great distances.

Neglecting the change in passing from the surface to the center, the formula is $s^3 = \dfrac{gr^2t^2}{2\pi^2}$. (See *Watson's Theoretical Astronomy*, page 46, eq. 28).

This formula gives a result somewhat greater than 90 radii of the earth. Assuming 90 radii as correct, the distance the spirits fell in the first second is,

$$x : 16.1 :: 1^2 : 90^2$$

x being .0237 inch.

Assuming that 460,000 radii was correct, the distance fallen in the first second would be .000,000,000,007,2 inch.

NOTE ON PROBLEM 95.

BY FLORIAN CAJORI, PH. D., PROFESSOR OF MATHEMATICS, COLORADO COLLEGE, COLORADO SPRINGS, COLO.

The problem arose in a discussion carried on in the *Nation*, Vol. 68, page 376, between Mr. C. S. Peirce and myself, relating to the validity of an argument given by Galileo and intended to refute the hypothesis that the velocity of a falling body varies as the distance described from a state of rest. Galileo says: "If the velocity with which a body overcomes four yards is double the velocity with which it passed over the first two yards, then the times necessary for these processes must be equal; but four yards can be overcome in the same time as two yards only if there is an instantaneous motion." Mr. Peirce argues that Galileo's reasoning is sound, a claim which I cannot admit.

The assumption that the velocity shall be proportional to the distance described from the state of rest can be expressed by the formula

$$\frac{ds}{dt}=as, \text{ where } a \text{ is a constant.}$$

Hence the acceleration is $\frac{d^2s}{dt^2}=a^2s$. Now initially the distance passed over is zero, *i. e.* $s=0$. Hence the initial accelleration is zero and the body can never begin to move. This conclusion stands even when a is infinitely large, for when absolute zero is a factor, then the product must be zero, no matter how large the other factor may be.* This is the point on which the whole discussion turns. Since Galileo concludes that instantaneous motion is the result, when really there can be no motion at all, his reasoning is fallacious.

But Peirce argues that Galileo used both assumptions stated in the problem, viz., (1) $\frac{ds}{dt}=as$, and (2) t finite for a finite distance. Peirce says: ". . . . the solution of the differential equation $\frac{ds}{dt}=as$ is $s=Ce^{at}$. In order that s and t should both be zero together, C must be infinitesimal. Then, for a finite value of s, either a or t must be infinite. That is, either the acquired velocity or the time of fall must be infinite. Galileo's argument first adduces the fact that the time is finite, and on that assumption concludes that the hypothesis would in-

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*"In putting together *naughts* to arrive at 1, *we never make any way at all;* the second thousand processes gives no more than the first." DE MORGAN.